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OPERATION OF AN INTEGRATING FLOATED-TYPE GYROSCOPE IN THE CAPACITY OF AN ANGULAR-MOTION NONFOLLOW-UP PICKUP

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Analysis of the error due to the device's deflection from zero point, arising when an integrating floated-type gyroscope is used, without a follow-up system, as an angular motion pickup. Suggestions are made concerning the use of such gyroscopes as pickups. Author

During operation of an integrating floated-type gyroscope with a servo, the angle of deflection of the gyro unit of the instrument from the neutral position usually does not exceed 0.001 radian. When the instrument works without a servo, this angle may reach several degrees, which introduces errors into the instrument readings.

The work of an integrating floated-type gyroscope as an angle of rotation pickup of an object in inertial space without a servo can be characterized by the equation

$$T\ddot{U} + \dot{U} = K_{\omega,\dot{U}}\left(1 + \frac{\Delta'_{\beta}}{K_{\beta,U}}\right)\left(\omega\cos\beta - \omega_{z0}\sin\beta - \right.$$
$$\left. - k_{f,\omega} f - k_{\ddot{\gamma},\omega}\ddot{\gamma} - \omega_{d}\right) \pm \Delta_{\dot{U}}. \quad (1)$$

In this equation:

$\dot{U}$ and $\ddot{U}$ = velocity and acceleration of the change of the output voltage U of the angular motion pickup;

ω = input angular velocity, i.e., absolute angular velocity of rotation of the instrument about its input axis;

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* Numbers in the margin indicate pagination in the original foreign text.

ω_{z0} = absolute angular velocity of the instrument case about the transverse axis z_0 perpendicular to the input and output axes of the instrument;

I = control current (current supplied to the control coil of the master);

$\ddot{\gamma}$ = absolute angular acceleration of the instrument case about the output axis;

β = angle of rotation of the gyro unit relative to the instrument case from the starting position corresponding to zero values of ω and U;

K and k = transmission coefficients (amplification factors) both of the instrument as a whole and of its individual components (the first index for the coefficient denotes the input quantity and the second, the output);

$$T = \frac{J}{K_{\dot{\beta},M_d}},$$

where: /84

T = time constant of instrument;

J = moment of inertia of the gyro relative to the output axis;

$K_{\dot{\beta},M_d}$ = specific damping moment;

$\dot{\beta}$ = angular velocity of the gyro about the output axis with respect to the instrument case;

M_d = damping moment;

$K_{\beta,U}$ = response (curvature of response curve) of angle sensor;

$$K_{\omega,U} = K_{\omega,\beta} K_{\beta,U}$$

= transmission coefficient of instrument with respect to input angular velocity;

$$K_{\omega, I} = \frac{H}{K_{I, M_{cd}}}$$

= transmission coefficient of the gyro;

H = intrinsic moment of the gyro;

Δ'_t and Δ'_β = partial derivatives of the first order of the absolute error Δ of the angle sensor with respect to time t and angle β, respectively (Bibl.1);

$$k_{I,\omega} = \frac{K_{I, M_{cd}}}{H},$$

where:

$K_{I, M_{cd}}$ = specific moment (curvature of the performance curve) of the moment sensor (master);

M_{Id} = moment produced by master when fed with a current I.

$$k_{\ddot{\gamma},\omega} = \frac{J}{H}.$$

The coefficients $k_{I,\omega}$ and $k_{\ddot{\gamma},\omega}$ characterize the effect on the instrument of, respectively, the current I and the angular acceleration $\ddot{\gamma}$. In this case, the coefficient $k_{I,\omega}$ is numerically equal to the angular velocity ω, having the same effect on the instrument as the single current I. Analogously, the coefficient $k_{\ddot{\gamma},\omega}$ is equal to the angular velocity ω, which has the same effect on the instrument as its spin with the single angular velocity $\ddot{\gamma}$.

$$\omega_d = \frac{M}{H}$$

= angular drift velocity (M = moment of noise acting about the spin axis of the gyro).

The instrument designed to operate without a servo should have an especially precise angle sensor so that its error Δ will be practically equal to zero. For an instrument with such a sensor, at $\omega_{z0} = 0$, $\cos \beta \approx 1$, and zero initial conditions, the solution of eq.(1) can be presented in the form /85

$$U = K_{\omega,U}\int_0^t \omega dt - K_{\omega,U}\left[\int_0^t\left(\omega_d + k_{I,\omega} I + k_{\ddot{\gamma},\omega}\ddot{\gamma}\right)dt + \right.$$
$$\left. + e^{-\frac{t}{T}}\int_0^t\left(\omega - \omega_d - k_{I,\omega} I - k_{\ddot{\gamma},\omega}\ddot{\gamma}\right)e^{\frac{t}{T}}dt\right]. \qquad (2)$$

In this expression, the first term of the right-hand side represents the output signal of an ideal instrument. Let us denote this signal by U_i. Since $\omega \equiv \alpha$, where α is the angle of rotation of the instrument case about the input axis relative to the inertial space during the time t, we have

$$U_i = K_{\omega,U}\alpha. \qquad (3)$$

The expression in brackets of the equality (2), for I = 0, gives the absolute error of measurement of the angle α, expressed in radians. In principle, this error can be compensated by a suitable current I. At ω = const, ω_d = = const, and $\ddot{\gamma}$ = const, the limiting value of this error will be

$$\Delta\alpha_l = -\left(\omega_d + k_{\ddot{\gamma},\omega}\ddot{\gamma}\right)(t - T) - \omega T.$$

To elucidate the effect of the time constant T on the accuracy of operation of the instrument, we will set $\omega_d = I = \ddot{\gamma} = 0$ in eq.(2) and will consider ω an arbitrary function of time. Then, the output voltage of the instrument becomes

$$U = K_{\omega,U}\int_0^t \omega dt - K_{\omega,U}e^{-\frac{t}{T}}\int_0^t \omega e^{\frac{t}{T}}dt.$$

Here, the first term of the right-hand side determines the output signal of the ideal instrument. The second term represents the absolute error ΔU_T of the instrument, resulting from $T \neq 0$. The corresponding relative error is

$$\varepsilon_T = \frac{\int_0^t \omega e^{\frac{t}{T}}dt}{e^{\frac{t}{T}}\int_0^t \omega dt}.$$

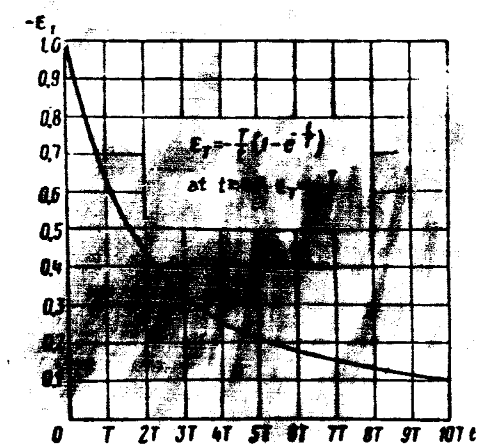

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Fig.1 Dependence of the Relative Error ε_T on the Time t

At ω = const,

$$\left.\begin{aligned} \Delta U_T &= -TK_{\omega,\dot{U}}\,\omega\left(1-e^{-\frac{t}{T}}\right), \\ \varepsilon_T &= -\frac{T}{t}\left(1-e^{-\frac{t}{T}}\right). \end{aligned}\right\} \quad (4)$$

In this case, the limiting value of the absolute velocity of the instrument will be

$$\Delta U_{Tl} = -TK_{\omega,\dot{U}}\,\omega$$

and is practically achieved at t = 3T. The limiting value of the relative error ε_T is equal to zero. The curve of the time dependence of ε_T constructed from eq.(4) is plotted in Fig.1.

If, for $t = t^*$, the angular velocity ω becomes equal to zero, then, for $t \geq t^*$, the output voltage U for $I = \ddot{\gamma} = \omega_d = 0$ will vary according to the law

$$U = K_{\omega,\dot{U}}\int_0^{t^*}\omega dt - K_{\omega,\dot{U}}\,e^{-\frac{t}{T}}\int_0^{t^*}\omega e^{\frac{t}{T}}\,dt$$

and, at $t \to \infty$, it will tend to the value which would occur at the instant of time $t = t^*$ when T = 0. This is shown graphically in Fig.2 where U^* is the output voltage of the instrument at the instant of time $t = t^*$ when $T \neq 0$, /87 while U_i^* is the same when T = 0.

The line OA characterizes the dependence of U on t at ω = const, for an instrument for which T = 0. The line OBC shows the dependence of U on t at

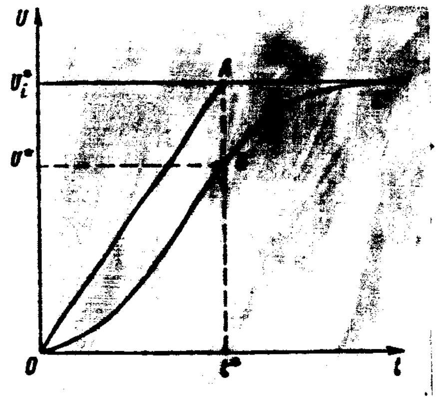

Fig.2 Transient Responses of Integrating Floated-Type Gyroscope at Rotation of the Instrument Case about the Input Axis with an Angular Velocity ω = const during the Time t^*

T $\neq$ 0. The segment OB corresponds to rotation of the instrument with a velocity ω = const, while the segment BC refers to the same, at a velocity ω = 0.

For instruments intended to operate without a servo, the time constant T should be no greater than for instruments working with a servo; therefore, for practical purposes we can consider T = 0 for such instruments.

To estimate the effect of the angle β and the angular velocity ω_{zo} on the operation of the instrument, we will assume that ω = const, ω_{zo} = const, T = = $\Delta = \omega_d = I = \ddot{\gamma} = 0$. Then, from eq.(1) we derive that, for zero initial conditions,

$$U = K_{\beta, U}\left(\sin^{-1}\frac{Ne^{qt}-1}{Ne^{qt}+1} - \varphi\right), \tag{5}$$

where

$$N = \left(n + \sqrt{1+n^2}\right)^2, \quad n = \frac{\omega_{z0}}{\omega},$$

$$q = 2K_{\omega,\beta}\sqrt{1+n^2}, \quad \tan^{-1}\varphi = n.$$

Thus, in the case under consideration the dependence of the voltage U on the angle α becomes nonlinear, which should be taken into account when calibrating the instrument. In this case, the nonlinearity of the output signal will be

$$\varepsilon_U = \frac{\Delta U}{U_i} = \frac{2\sqrt{1+n^2}}{q\alpha}\left(\sin^{-1}\frac{Ne^{q\alpha}-1}{Ne^{q\alpha}+1}-\varphi\right)-1, \tag{6}$$

where

U_i = output voltage for $\beta \approx 0$ and $\omega_{zo} = 0$ determined by eq.(3);

$$\Delta U = U - U_i;$$

U = actual output voltage determined by the equality (5).

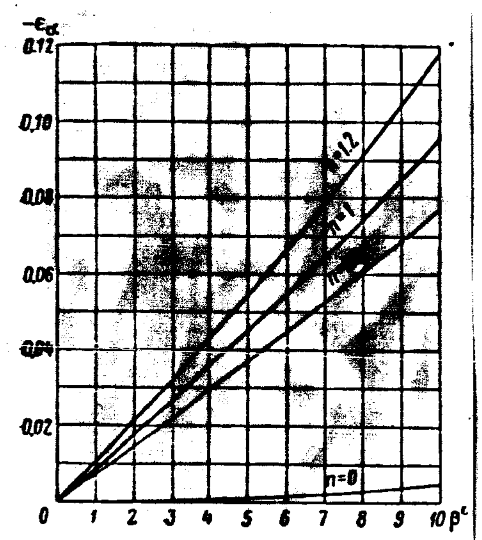

Fig.3 Dependence of the Relative Error ϵ_α on the Angle β, at Various Values of $n = \frac{\omega_{zo}}{\omega}$

The maximum nonlinearity will occur at $\beta = \beta_{max}$. For its determination, we must set in the expression for ϵ_U

$$q\alpha = (q\alpha)_{max} = \ln\frac{1+\sin(\beta_{max}+\varphi)}{N[1-\sin(\beta_{max}+\varphi)]} \tag{7}$$

In the case in question, the dependence between the angles α and β is /88
determined by the equality

$$\alpha = \frac{1}{2K_{ш.р}\sqrt{1+n^2}}\ln\frac{1+\sin(\beta+\varphi)}{N[1-\sin(\beta+\varphi)]} \tag{8}$$

When calibrating the instrument with respect to eq.(3), the relative error of measuring the angle α, produced by the effect of the angle β and the angular velocity ω_{z0}, expressed in fractions of the true value of α, will read

$$\varepsilon_\alpha = \frac{2\beta\sqrt{1+n^2}}{\ln\dfrac{1+\sin(\beta+\varphi)}{N[1-\sin(\beta+\varphi)]}} - 1. \tag{9}$$

From this formula, curves (Fig.3) are plotted which show the dependence of ε_α on the angle β, for various values of n.

To derive the formulas characterizing the work of the instrument at any values of the angle β and at zero value of the angular velocity ω_{z0}, we set $n = \varphi = 0$ and $N = 1$ in eqs.(5) - (9).

The value of the angle α, shown by the instrument, is

$$\alpha_{inst.} = (1+\varepsilon_\alpha)\alpha.$$

When examining the dynamic properties of an integrating floated-type gyroscope, we can assume $\Delta'_\beta = \Delta'_t = 0$ by virtue of their smallness. Then, postulating that the angle β does not exceed 10° and thus taking $\cos\beta \approx 1$, we find from eq.(1) that, at $\omega_{z0} = 0$, the transmission coefficients, the transfer functions, and the amplitude and phase frequency responses of the integrating floated-type gyroscope with respect to ω, I, $\ddot{\gamma}$, and M will have the values shown in the Table.

The integrating floated-type gyroscope, designed to operate without a servo, i.e., intended for direct use as a sensor of the angle of rotation of an object relative to inertial space should, unlike the instrument working with a servo, have the smallest possible transmission coefficient of the gyro $K_{\omega\dot{\beta}}$. In this /90
case, the required value $K_{\omega\dot{\beta}}$ is determined by the equality

$$K_{\omega\dot{\beta}} = \frac{\beta_{max}}{\alpha_{max}},$$

TABLE

TRANSMISSION COEFFICIENTS, TRANSFER FUNCTIONS, AND AMPLITUDE AND PHASE FREQUENCY RESPONSES OF AN INTEGRATING FLOATED-TYPE GYROSCOPE WITH RESPECT TO VARIOUS INPUT QUANTITIES

Input Quantity	Transmission Coefficient	Transfer Function $W(p)$	Amplitude Frequency Response A	Phase Frequency Response B
Input angular velocity ω	$K_{\omega.\dot{U}} = \frac{HK_{\beta.U}}{K_{\dot{\beta}.M}}$	$\frac{K_{\omega.\dot{U}}}{p(Tp+1)}$	$\frac{K_{\omega.\dot{U}}}{2\pi f\sqrt{(2\pi fT)^2+1}}$	$-\frac{\pi}{2}-\tan^{-1}(2\pi fT)$
Control current I	$K_{I.\dot{U}} = \frac{-K_{I.M_3}K_{\beta.U}}{K_{\dot{\beta}.M_d}}$	$\frac{K_{I.\dot{U}}}{p(Tp+1)}$	$\frac{\lvert K_{I.\dot{U}}\rvert}{2\pi f\sqrt{(2\pi fT)^2+1}}$	
Absolute angular velocity $\dot{\gamma}$ of the body of the device about the output axis	$K_{\ddot{\gamma}.\dot{U}} = \frac{-JK_{\beta.U}}{K_{\dot{\beta}.M_d}}$	$\frac{K_{\ddot{\gamma}.\dot{U}}}{p(Tp+1)}$	$\frac{\lvert K_{\ddot{\gamma}.\dot{U}}\rvert}{2\pi f\sqrt{(2\pi fT)^2+1}}$	$\frac{\pi}{2}-\tan^{-1}(2\pi fT)$
Moment of noise M	$K_{M.\dot{U}} = \frac{-K_{\beta.U}}{K_{\dot{\beta}.M_d}}$	$\frac{K_{M.\dot{U}}}{p(Tp+1)}$	$\frac{\lvert K_{M.\dot{U}}\rvert}{2\pi f\sqrt{(2\pi fT)^2+1}}$	

where α_{max} is the maximum value of the measured angle α, i.e., the angle of rotation of the instrument about its input axis; β_{max} is the maximum permissible value of the angle of rotation of the gyro corresponding to the angle α_{max} and selected on the basis of the permissible error ϵ_α, resulting from the effect of the angle β and the angular velocity ω_{z0} determined by eq.(9).

The realization of small values of $K_{\omega\beta}$ requires the realization of quite large values of $K_{\dot{\beta},M_d}$, which meets with considerable technical difficulties. To be able to measure rather large values of the angle α, it is necessary to decrease $K_{\omega\dot{\beta}}$ and to increase β_{max}. As a consequence of this, the instruments working without a servo should have, in comparison with instruments working with a servo, a considerably lower value of $K_{\omega\dot{\beta}}$ and a much larger maximum angle of deflection of the gyro from its initial position.

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